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The Origin of Warped, Precessing Accretion Disks in X-Ray Binaries

Philip R. Maloney¹ and Mitchell C. Begelman^{2,3}

ABSTRACT

The radiation-driven warping instability discovered by Pringle holds considerable promise as the mechanism responsible for producing warped, precessing accretion disks in X-ray binaries. This instability is an inherently *global* mode of the disk, thereby avoiding the difficulties with earlier models for the precession. Here we follow up earlier work to study the linear behavior of the instability in the specific context of a binary system. We treat the influence of the companion as an orbit-averaged, quadrupole torque on the disk. The presence of this external torque allows the existence of solutions in which the direction of precession of the warp is retrograde with respect to disk rotation, in addition to the prograde solutions which exist in the absence of external torques.

Subject headings: accretion disks – instabilities – stars: individual (Her X-1, SS 433) – X-rays: stars

1. Introduction

For a quarter of a century, evidence has been accumulating for the existence of warped, precessing disks in X-ray binary systems. The discovery of a 35-day period in the X-ray flux from Her X-1 (Tananbaum *et al.* 1972) was interpreted almost immediately as the result of periodic obscuration by a precessing accretion disk which is tilted with respect to the binary plane (Katz 1973). Katz proposed that the precession was forced by the torque from the companion star, but left unexplained the origin of the disk’s misalignment. An alternative possibility is the “slaved disk” model of Roberts (1974): here it is actually the companion star that is misaligned and

¹Center for Astrophysics and Space Astronomy, University of Colorado, Boulder, CO 80309-0389; maloney@shapley.colorado.edu

²JILA, University of Colorado and National Institute of Standards and Technology, Boulder, CO 80309-0440; mitch@jila.colorado.edu

³Also at Department of Astrophysical and Planetary Sciences, University of Colorado

precessing; the accretion disk (fed by the companion) will track the motion of the companion provided the residence time in the disk is sufficiently short.

Dramatic evidence for a warped, precessing disk in an X-ray binary was provided by the discovery of the relativistic precessing jets in SS 433 (Margon 1984, and references therein). The systematic velocity variations of the optical jet emission, the radio jet morphology, and optical photometry of the system all indicate a precession period for the disk of 164 days; this must be a *global* mode, as both the inner disk (to explain the jets) and the outer disk (to explain the photometry) must precess at the same rate. The systematic variation of the X-ray pulse profile of Her X-1 has been interpreted as the result of precession of the inner edge of an accretion disk, which argues for a global mode in this object also (Trümper *et al.* 1986; Petterson, Rothschild, & Gruber 1991). A crucial point is that in both Her X-1 and SS 433, the direction of precession of the warp has been inferred to be retrograde with respect to the direction of rotation of the disk (*e.g.*, Gerend & Boynton 1976 for Her X-1; Leibowitz 1984 and Brinkmann, Kawai & Matsuoka 1989 for SS 433).

A number of other X-ray binaries, both high and low mass, show evidence for long period variations which may indicate the presence of precessing, inclined disks: LMC X-4 (30.5 days), Cyg X-1 (294 days), XB1820-303 (175 days), LMC X-3 (198 or 99 days), and Cyg X-2 (77 days) (Priedhorsky & Holt 1987, and references therein; Cowley *et al.* 1991; Smale & Lochner 1992; White, Nagase, & Parmar 1995). Given the small number of X-ray binaries for which adequate data exist to test for the existence of such periodicities, it is evident that precessing, inclined accretion disks may be common in X-ray binaries.

Although theoretical attempts to understand the mechanism responsible for producing precessing, tilted or warped accretion disks date from the discovery of Her X-1, no generally accepted model has emerged. The original models suggested for Her X-1 suffer from serious flaws, although both predict retrograde precession. The model of Katz (1973) imposes a tilted disk as a boundary condition, but provides no mechanism for producing this tilt. The slaved disk model (Roberts 1974) requires that the companion star rotation axis be misaligned with respect to the binary plane; however, the axial tilt is expected to decay by tidal damping on a timescale shorter than the circularization time (Chevalier 1976). Other suggested mechanisms suffer from the difficulty of communicating a single precession frequency through a fluid, differentially-rotating disk (*e.g.*, Maloney and Begelman 1997).

Recently, however, a natural mechanism for producing warped accretion disks has been discovered. Motivated by work by Petterson (1977) and Iping & Petterson (1990), Pringle (1996) showed that centrally-illuminated accretion disks are unstable to warping due to the pressure of re-radiated radiation which, for a non-planar disk, is non-axisymmetric and therefore exerts a torque. Further work on Pringle’s instability was done by Maloney, Begelman, & Pringle (1996), who obtained exact solutions to the linearized twist equations, by Maloney, Begelman, & Nowak (1997) (hereafter MBN), who generalized the earlier work to consider non-isothermal disks (see

below), and by Pringle (1997), who examined the nonlinear evolution. Radiation-driven warping is an inherently global mechanism; the disk twists itself up in such a way that the precession rate is the same at each radius.

Previous work on Pringle’s instability assumed no external torques. However, in X-ray binary systems the torque exerted on the accretion disk by the companion star must dominate at large radii. In the present paper we examine the behavior of the instability when an external torque is included. In §2 we derive the modified twist equation and solve it numerically, and in §3 we discuss the solutions and the implications for X-ray binaries.

2. The Twist Equation and Solutions

We use Cartesian coordinates with the Z -axis normal to the plane of the binary system; hence $Z = 0$ is the orbital plane. As in MBN, we assume that the disk viscosity $\nu_1 = \nu_o (R/R_o)^\delta$, where R_o is an arbitrary fiducial radius. For a steady-state disk far from the boundaries, this implies that the disk surface density $\Sigma \propto R^{-\delta}$. In the α -viscosity prescription, $\delta = 3/2$ corresponds to an isothermal disk. We distinguish between the usual azimuthal shear viscosity ν_1 and the vertical viscosity ν_2 which acts on out-of-plane motions (see *e.g.*, Papaloizou & Pringle 1983, Pringle 1992); the ratio $\nu_2/\nu_1 \equiv \eta$ is assumed to be constant but not necessarily unity. Assuming an accretion-fueled radiation source, we transform to the radius variable $x = (\sqrt{2}\epsilon/\eta) (R/R_s)^{1/2}$, where $\epsilon \equiv L/\dot{M}c^2$ is the radiative efficiency and R_s is the Schwarzschild radius. The linearized equation governing the disk tilt (including radiation torque) for a normal mode with time-dependence $e^{i\sigma t}$ is then

$$x \frac{\partial^2 W}{\partial x^2} + (2 - ix) \frac{\partial W}{\partial x} = i\tilde{\sigma} x^{3-2\delta} W. \quad (1)$$

(eq. [10] of MBN). The tilt is described by the function $W \equiv \beta e^{i\gamma}$, where β is the local tilt of the disk axis with respect to the Z -axis and γ defines the azimuth of the line of nodes. We have nondimensionalized the eigenfrequency σ by defining $\tilde{\sigma} \equiv 2\eta^3 R_s^2 \sigma / \epsilon^4 \nu_o$; in general $\tilde{\sigma}$ is complex, with real part $\tilde{\sigma}_r$ and imaginary part $\tilde{\sigma}_i$. Negative values of $\tilde{\sigma}_i$ correspond to growing modes. We also define R_o such that $x(R_o) = 1$; therefore $R_o = (1/2)(\eta/\epsilon)^2 R_s$.

Equation (1) assumes that the only torques are produced by viscous forces and radiation pressure. In this case, the precession of the warping modes *must* be prograde, *i.e.*, in the same direction as disk rotation (MBN). In a binary system, however, the companion star also exerts a force on the accretion disk. Since the orbital period is much shorter than the viscous timescale, we average the resulting torque over azimuth and keep only the leading (quadrupole) terms (*e.g.*, Katz *et al.* 1982). This orbit-averaged torque term will cause a ring of the disk that is tilted with respect to the binary plane to precess, in a retrograde sense, about the normal to the binary plane. (Including the time dependence of the torque would result in nutation as well as precession, a complication we do not consider here.) Relative to the angular momentum density of a ring,

$\Sigma R^2 \Omega$, the torque term is

$$\frac{\mathbf{T}}{\Sigma R^2 \Omega} = \frac{3}{8} \left(\frac{GM}{r^3} \right)^{1/2} \left(\frac{R}{r} \right)^{3/2} \left(\frac{M}{M_x} \right)^{1/2} \sin 2\beta (\sin \gamma, -\cos \gamma, 0) \quad (2)$$

where R is the distance from the compact object of mass M_x and r is the separation between the companion star of mass M and the compact object. Writing the R -dependent coefficient of this term as $\omega_o (R/R_o)^{3/2}$, the quadrupole torque then contributes a term $i\tilde{\omega}_o x^3 W$ to the righthand side of the twist equation (1), so that the equation becomes

$$x \frac{\partial^2 W}{\partial x^2} + (2 - ix) \frac{\partial W}{\partial x} = ix^{3-2\delta} (\tilde{\sigma} + \tilde{\omega}_o x^3) W, \quad (3)$$

where the quadrupole precession frequency $\tilde{\omega}_o$ has been nondimensionalized in the same manner as the eigenfrequency in equation (1).

Equation (3) must be solved numerically. However, an entire class of solutions can be derived from the results of MBN, which do *not* include an external torque. Comparison of the twist equation with and without quadrupole torque shows that equation (3) with $\tilde{\sigma} = 0$ is formally identical to equation (1) (in which $\tilde{\omega}_o = 0$) if we replace δ with $\delta' = \delta - 3/2$. In other words, the purely precessing (*i.e.*, real $\tilde{\sigma}$, since $\tilde{\omega}_o$ is real) modes with no external torque have the same shapes as non-precessing modes *with* quadrupole torque, with $\tilde{\omega}_o = \tilde{\sigma}_r$, except that the latter correspond to a larger value of the surface density index δ . The fact that these modes are non-precessing, *i.e.*, the warp shape is fixed in an arbitrary inertial frame, is rather remarkable, as it requires that the radiation torque (which attempts to make the warp precess in a prograde sense) and the quadrupole torque (which attempts to make the disk precess in the retrograde direction) precisely balance at each radius.

We impose as the outer boundary condition that the disk must cross the $Z = 0$ plane at some radius, and impose the usual no-torque inner boundary condition. This choice of $Z = 0$ for the outer boundary condition is not strictly correct; what we take as the disk boundary here corresponds to the circularization radius, R_{circ} , in a real X-ray binary, and the actual outer boundary will typically be at $R_{\text{out}} \sim 3R_{\text{circ}}$. The proper boundary condition then constrains the gradient $RW^{-1}\partial W/\partial R$ at R_{circ} (J. Pringle 1997, private communication). We have modelled the outer disk ($R_{\text{circ}} \leq R \leq R_{\text{out}}$), assuming that $V_R = 0$ for $R \geq R_{\text{circ}}$ and $W' = 0$ at R_{out} , and have examined how these solutions couple to the inner disk solutions. We find that the gradient is always steep at R_{circ} , so that the outer and inner disk solutions match up at radii that differ by $\lesssim 10\%$ from the radius of the zero, and the tilt declines rapidly to zero for $R > R_{\text{circ}}$. Thus, the true solutions differ little from the zero-crossing eigenfunctions calculated here, and we can ignore any minor differences for the purposes of this paper. A full discussion of this important point is given in MBN.

As in MBN, we separate equation (3) into real and imaginary parts and solve as an initial-value problem, iterating to find the zero-crossing eigenfunctions. In the absence of any external torque, there is a marked change in the behavior of the eigenfunctions across $\delta = 1$ (see MBN), so

here we consider the two cases $\delta = 0.75$ (corresponding to the usual gas pressure-supported Shakura-Sunyaev disk) and $\delta = 1.25$.

In Figure 1 we plot the location x_o where the eigenfunction returns to the $Z = 0$ plane (in the terminology of MBN, these are the first-order zeros, closest to the origin) and the normalized precession rate $\tilde{\sigma}_r$, as a function of the dimensionless quadrupole frequency $\tilde{\omega}_o$, for $\delta = 0.75$; Figure 2 plots the same quantities for $\delta = 1.25$. The series of curves are for different growth rates, from a value of $\tilde{\sigma}_i$ close to the maximum growth rate, down to small growth rates for which the curves essentially coincide with the steady-state $\tilde{\sigma}_i = 0$ curve, not shown. The dotted portions of the curves mark the prograde modes, the solid portions show the retrograde modes. Although there are significant differences in behavior for the two values of δ , there are overall similarities: (1) There is a maximum value of $\tilde{\omega}_o$ above which the torque from the companion is too large to allow warped modes to exist; (2) For a given value of $\tilde{\omega}_o$ the solutions are generally double-valued, with one prograde and one retrograde mode or two prograde modes; the retrograde modes return to the plane at larger radius than the prograde modes; (3) the precession rates of the modes $\tilde{\sigma}_r$ are much larger than the associated values of $\tilde{\omega}_o$; this is unsurprising since the quadrupole torque fixes the boundary condition at the disk’s *outer* edge, which is always at $R \gg R_o$ (recall $x(R_o) \equiv 1$).

For any single value of the growth rate $\tilde{\sigma}_i$, the disk boundary x_o is at a fixed radius for a given value of $\tilde{\omega}_o$ (and choice of solution in the double-valued regime). The value of $\tilde{\omega}_o$, in turn, is set by the masses and separation of the components of the binary. In general this value of x_o will not match the actual outer boundary of the disk. This implies that in real disks, the location of the outer boundary will determine the growth rate, provided that the outer boundary falls within the range of radii occupied by the warp modes, either prograde or retrograde. As can be seen from Figures 1a and 2a, the fraction of the $(\tilde{\omega}_o, x_o)$ –plane occupied by retrograde modes is non-negligible, although it is smaller than that for prograde modes. The retrograde modes essentially always have their outer boundaries at larger radius than the prograde modes. The precession rates $\tilde{\sigma}_r$ are usually comparable for the prograde and retrograde solutions, although for $\delta = 1.25$, $|\tilde{\sigma}_r|$ for the retrograde modes may be several times larger than for the prograde modes. (The abrupt termination of the curves for these first-order eigenfunctions is genuine; the higher-order eigenfunctions extend to larger x , generally with different values of $\tilde{\sigma}_r$ and $\tilde{\sigma}_i$ for a given value of $\tilde{\omega}_o$.)

3. Discussion

Where do real X-ray binary systems lie in this parameter space? In order to quantify our results, we must make a specific assumption about the viscosity; we also evaluate the results for $\delta = 0.75$, but there is in fact very little dependence on δ . The precession rates can be expressed in terms of the viscous timescale $\tau_{\text{visc}} \sim R/V_R \sim 2R^2/3\nu_1$ at the critical radius R_{cr} (the minimum

radius for instability: typically $R_{\text{cr}} \sim 10^9 \eta^2 (0.1/\epsilon)^2 (M/M_\odot)$ cm; see MBN), with $x_{\text{cr}} \equiv x(R_{\text{cr}})$:

$$\sigma_r = \frac{\eta x_{\text{cr}}^{4-2\delta}}{12\tau_{\text{visc}}(R_{\text{cr}})} \tilde{\sigma}_r. \quad (4)$$

Using the α -prescription for viscosity, defining the precession timescale to be $\tau_{\text{prec}} \equiv 2\pi/\sigma_r$, and normalizing to typical values, we find

$$\tau_{\text{prec}} \sim 25 \frac{\eta^2}{(\epsilon/0.1)^3} \frac{M/M_\odot}{(\alpha/0.1)} \left(\frac{H/R}{0.01}\right)^{-2}_{R_{\text{cr}}} \left(\frac{\tilde{\sigma}_r}{0.1}\right)^{-1} \text{ days}. \quad (5)$$

Thus the expected precession timescales for the disks are weeks to months.

The quadrupole precession frequency ω_o is given by

$$\begin{aligned} \omega_o &= \frac{3}{4} \left(\frac{GM}{r^3}\right)^{1/2} \left(\frac{R_o}{r}\right)^{3/2} \left(\frac{M}{M_x}\right)^{1/2} \\ &\simeq 4.9 \times 10^{-13} \left(\frac{\eta}{\epsilon}\right)^3 \frac{(M/M_\odot)^2}{r_{11}^3} \left(\frac{M}{M_x}\right)^{1/2} \text{ s}^{-1} \end{aligned} \quad (6)$$

where $r_{11} = r/10^{11}$ cm. Since $\tilde{\omega}_o$ is normalized in the same manner as $\tilde{\sigma}$, we find

$$\tilde{\omega}_o \sim 2 \times 10^{-5} \frac{\eta^5}{\epsilon_{0.1}^6} \frac{(M/M_\odot)^3}{(\alpha/0.1)r_{11}^3} \left(\frac{M}{M_x}\right)^{1/2} \left(\frac{H/R}{0.01}\right)^{-2}_{R_{\text{cr}}}. \quad (7)$$

From Figure 1, it is evident that the relevant values of $\tilde{\omega}_o$ fall in the range where warped disk solutions exist, with either prograde or retrograde precession. Furthermore, the outer boundary radius x_o which characterizes these solutions is typically $R_{\text{out}} \sim 10^5 R_s$, which is also the expected value for real X-ray binary systems.

The estimates of τ_{prec} and $\tilde{\omega}_o$ are sensitive to the value of the radiative efficiency ϵ , extremely so in the latter case. However, we do not expect this to be true in real X-ray binary systems. The steep dependence on ϵ in the linear theory estimate comes from the scaling of the fiducial radius R_o . However, physically, the important quantity will be the value of ω at the disk boundary, not its value at R_o , since it is the outer boundary condition which determines the importance of the torque from the companion. Thus we expect that in reality the timescales and frequencies will be less sensitive to ϵ than the above estimates.

The linear theory of disk warping leaves a number of questions unanswered. Although we have demonstrated that retrograde as well as prograde solutions exist when a quadrupole torque is included, there does not appear to be any reason for choosing one mode over another. For a fixed value of ω_o , the retrograde solutions (when both exist) occur for larger values of the boundary radius, but this difference is not large, only a factor of $\sim 5 - 10$ at most (see Figures 1 and 2). In Figure 3 (Plate 1) we show the shapes of the most rapidly rotating prograde and retrograde modes for $\delta = 1.25$, for both the steady-state and fastest-growing modes. (The $\delta = 0.75$ modes are very similar; this difference from the solutions with no external torque [MBN], in which the shapes of

the growing modes are very different for $\delta > 1$ and $\delta < 1$, is a reflection of the importance of the quadrupole torque, which always dominates at large radius.) The fast-growing prograde modes are more “wound up” (*i.e.*, the azimuth of the line of nodes rotates through a larger angle between the origin and the disk boundary) than the fast-growing retrograde modes, which may result in differences in the effects of self-shadowing on the disk. An important observational task, given our earlier argument that warped, precessing disks are probably common in X-ray binaries, will be to determine if retrograde precession is the norm in such systems, or only some fraction of them. As we noted earlier, both of the original suggestions for producing precessing disks in X-ray binaries require that the sense of precession be retrograde.

We also cannot establish from linear theory that steady, long-lived solutions actually exist. Work on the nonlinear evolution of radiation-warped disks (with no external torques) by Pringle (1997) suggests that chaotic behavior can result, in consequence of the feedback between disk shadowing and the growth of the warp. Calculation of the nonlinear evolution of disks in X-ray binaries will be necessary to determine whether steady solutions do exist in this case, and if so under what conditions. We have also ignored the effects of winds, which could also drive warping (Schandl & Meyer 1994; Pringle 1996). However, in the case of X-ray heated winds, this seems unlikely to be important, since at the base of the wind the gas pressure is much less than the radiation pressure (*e.g.*, Begelman *et al.* 1983).

Pringle’s instability appears to be very promising as a solution to this long-standing problem. It is expected to be generically important in accretion disks around compact objects, and is inherently a *global* mode of the disk, so that the warp shape precesses with a single pattern speed. In addition, it provides a natural mechanism for producing non-planar disks in X-ray binary systems, so that the torque from the companion star (which affects only the out-of-plane portion of the disk) is able to influence the disk shape, allowing retrograde as well as prograde modes to exist.

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Fig. 1.— (a) (*Top*) The location of the disk outer boundary, x_o , as a function of the dimensionless quadrupole precession frequency $\tilde{\omega}_o$, for surface-density index $\delta = 0.75$. From right to left, the curves are for increasing values of the growth rate, $-\tilde{\sigma}_i$; the plotted curves are for $-\tilde{\sigma}_i = 0.001, 0.01, 0.02, 0.03, 0.04$, and 0.05 . The solutions with retrograde precession are marked with solid lines, the prograde with dotted lines. The maximum growth rate plotted, 0.05 , is very close to the maximum allowed growth rate. (b) (*bottom*) The dimensionless precession rate, $\tilde{\sigma}_r$, for the same models as in (a).

Fig. 2.— (a) (*Top*) As in Figure 1a, for $\delta = 1.25$. From right to left, the plotted curves are for $-\tilde{\sigma}_i = 0.01, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$, and 1.0 . The maximum growth rate plotted, 1.0 , is very close to the maximum allowed growth rate. (b) (*bottom*) The dimensionless precession rate, $\tilde{\sigma}_r$, for the same models as in (a).

Fig. 3.— Surface plots showing the shapes of several of the warped disk modes, for $\delta = 1.25$. In all cases the amplitude of the warp has been fixed at 20%, and the solutions have been plotted to the disk boundary. *Top left*: The most rapidly-rotating (largest σ_r) steady-state ($\sigma_i = 0$), prograde mode. *Top right*: The most rapidly-rotating, steady-state retrograde mode. *Bottom left*: The fastest-growing (maximum $-\sigma_i$), most rapidly-rotating prograde mode. *Bottom right*: The fastest-growing, most rapidly-rotating retrograde mode.





